1. (a) Let 'r' be the remainder ⇒ 221 – r, 116 – r, 356 – r are exactly divisible by that number. Now, if two numbers are divisible by a number, then so is their difference
⇒ [(221 – r) – (116 – r)], [(356 – r) – (116 – r)]. and [(356 – r) – (221 – r)] are divisible by that number
⇒ 105, 135, 240 are divisible by that number
= HCF of 105, 135, 140 = 15.

2. (a) On dividing the given number 345670 by 6, we get 4 as the remainder.
So 2 must be added to the given number.

3. (d) Since (35 – 25) = 10, (45 – 35) = 10, (55 – 45) = 10. Now take the LCM of 35, 45, 55 and subtract 10 from it
⇒ 3465 – 10 = 3455.

4. (c) $n(n^2 + 20)$ is always divisible by 24, if n is even number.

5. (a) When 2^{256} is divided by 17 then
$$\Rightarrow \frac{2^{256}}{2^4+1} \Rightarrow \frac{(2^2)^{64}}{(2^4+1)}$$
By remainder theorem when $f(x)$ is divided by $x + a$ the remainder = $f(-a)$
Here $f(1) = (2^2)^{64}$ and $x = 2^4$ and $a = 1$
∴ Remainder = $f(-1) = (-1)^{64} = 1$

6. (b) The last digit of 2137^1 is 7.
Last digit of 2137^2 is 9 .Last digit of 2137^3 is 3, the last digit of 2137^4 is 1, last digit 2137^5 is 7 and the last digit of 2137^6 is 9 and so on . Hence it form a pattern and the last digit repeats for every 5th . 753 = 4 × 188 + 1 . Thus the last digit of 2137^{753} is the same as that of 2137^1 *i. e.*, 7.

7. (a) We have to find the least number which is divisible by 3, 5, 6 and 9 and is also a perfect square. The LCM of 3, 5, 6 and 9 is 3 × 3 × 2 × 5 = 90. Hence, the required number is 90 × 2 × 5 = 900.

8 (c) Use test of 11 after putting $y = 5$.

9. (d) Out of n and $n + 2$, one is divisible by 2 and the other by 4, hence $n(n + 2)$ is divisible by 8. Also n, $n + 1$, $n + 2$ are three consecutive numbers, hence one of them is divisible by 3. Hence $n(n + 1)(n + 2)$ must be divisible by 24. This will be true for any even number n.

10. (b) Check the number for divisibility by 3.
So, 4 + 5 + 9 + 0 + 4 + 5 = 27. Hence it is divisible by 3 and the quotient is 153015.
Now, check the quotient for divisibility by 9.
1 + 5 + 3 + 0 + 1 + 5 = 15
So, the number is not divisible by 9.
However, if we add 3 to the number *i.e.*, 153015 + 3 = 153018 it would be, divisible by 9.
So, the number divisible by 27 will be –153015 + 3 × 3 = 459054 *i.e.*, 9 should be added.

11. (c) Last digit in 19 – 9
$18^2 - 1$
$19^3 - 1$
for odd powers of 19
Last digit is 9 and for even it is 1
Last digit in 19^{81} is 9
Last digit in 41 is 4
42 is 6
43 is 4
for odd powers of 4
3^9k is odd irrespective of the value of k
∴ last digit in 4^9k is 4. Last digit in $19^{81} + 4^9k$ is last digit in 9 + 4 *i.e*, in 13 = 3

12. (a) Sum of prime numbers that are greater than 60, but less than 70 is
61 + 67 = 128

13. (d) 311 is repeated seven times in the number, 311 is not divisible by 3 but 311 repeated twice is not divisible by 3, but divisible by 11.
Similarly 311 repeated thrice is divisible by 3, but not by 11.
As 311 is repeated seven times, which is neither multiple of 2 nor 3.
So, number is not divisible by 3 or 11.

14. (c) $\frac{1365-15}{5} = 270$

15. (c) Sum of digits = (5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x), which must be divisible by 3.
∴ $x = 2$.

16. (d) Clearly, 4864 is divisible by 4.
So, 9P2 must be divisible by 3, so, (9 + P + 2) must be divisible by 3.
∴ $P = 1$.

17. (a) Largest 4-digit number = 9999

```
88 ) 9999 ( 113
      88
     ---
      119
       88
     ---
      319
      264
     ---
       55
```

Required number = (9999 – 55) = 9944

18. (a) $(x^n - a^n)$ is always divisible by $(x + a)$, when n is even natural number.

19. (d) $0.\overline{47} = \frac{47}{99}$.

20. (c)

21. (b) LCM of the numbers = 420. Hence there must be $(420 \times 2) + 2 = 842$ beads.

22. (b) Since $59 = 4 \times 14 + 3 \Rightarrow$ last digit of $(377)^{59} = 3$
$87 = 4 \times 21 + 3 \Rightarrow$ last digit of $(793)^{87} = 7$
$129 = 4 \times 32 + 1 \Rightarrow$ last digit of $(578)^{129} = 8$
$99 = 2 \times 49 + 1 \Rightarrow$ last digit of $(99)^{59} = 9$
Hence the last digit of the result is equal to the last digit of
$3 \times 7 \times 8 \times 9$, *i.e.*, 2.
$\therefore$ digit at unit's place = 2

23. (d) Interval after which the devices will beep together = (L.C.M. of 30, 60, 90, 105) min = 1260 min. = 21 hrs.
So, the devices will again beep together 21 hrs. after 12 noon *i.e.*, at 9 a.m.

24. (d) N will be an odd number because N is sum of one even number (b) and 13985 odd numbers.
Hence, N will not be divisible by an even number.

25. (b) Divisor $= r_1 + r_2 - r_3 = 35 + 30 - 20 = 45$

26. (b) $12 - 7 = 5$, $15 - 10 = 5$ and $16 - 11 = 5$
Hence the desired number is 5 short for divisibility by 12, 15 and 16.
L.C.M. of 12, 15, 16 is 240
Hence the least number = 240 – 5 = 235

27. (c) We have to find numbers between 100 and 200 which are even and are neither divisible by 7 nor by 9.
$\therefore$ No. that are even and are divisible by 7 are 7 and no. which are even and divisible by 9 are 6.
Nos. even and divisible by 7 and 9 both are (*e.g.*, 63) is only 126 :
$\therefore$ Required answer = 7 + 6 – 1 = 12
$\therefore$ 51 – 12 = 39.

28. (a) Let the numbers be the form $10x + y$
According to question
$10x + y = x + y + xy$
$9x = xy$
$\therefore$ $y = 9$
The numbers are 19, 29, 39, 49, 59, 69, 79, 89 and 99 total of 9 numbers
Hence the required fraction $= \frac{9}{91}$
$= 0.099 \approx 0.1$

29. (b) Let there be w wide runs.
Byes = $w + 8$
Runs scored by batsmen = 26 w
Total run = 232
or $w + w + 8 + 26W = 323$
$\Rightarrow w = \frac{224}{28} = 8$
$\therefore$ Run scored by Ram $= \frac{6}{13} \times 208 = 96$

30. (b) The value of the expression will be least when $x = y = z = 1/3$.
Hence, the least value $= \left(\frac{1}{1/3} - 1\right)^3$
$= 2 \times 2 \times 2 = 8$.

31. (b) Consider $3^{4n} = (81)^n = (1 + 80)^n = 1 + 80q,\ q \in N$
$\therefore 3^{3^{4n}} = 3^{80q+1} = (81)^{20q} . 3$
Since the last digit of $(81)^{20q}$ is 1, so the last digit of $3^{3^{4n}} + 1$ is $1 \times 3 + 1 = 4$

32. (c) The last digit in the number must be 6: for only numbers ending in 6, when raised to any power, result in another no. ending in 6.

33. (a) Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.
Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.
Hence, digits in place of * and $ are 4 and 0 respectively.

34. (a) Let total number of seats in the stadium be p; number of seats in the lower deck be x and number of seats in upper deck be y.
$\therefore$ $p = x + y,\ x = p/4,\ y = 3p/4$
Now in the lower deck, $4x/5$ seats were sold and $x/5$ seats were unsold.
No. of total seats sold in the stadium = $2p/3$.
No. of unsold seats in the lower deck = $x/5 = p/20$
No. of unsold seats in the stadium = $p/3$
$\therefore$ Required fraction $= \frac{p/20}{p/3} = \frac{3}{20}$

35. (c) $1 + 2 + 3 + \ldots + 40 = \frac{40 \times 41}{2} = 820$
Since at each time any two numbers a and b are erased and a single new number $(a + b - 1)$ is writen. Hence, each one is subtracted and this process is repeated 39 times. Therefore, number left on the board at the end = 820 – 39 = 781.

36. (d) Since 80 = 8 × 10 or 80 = 16 × 5
Thus y (*i.e.*, unit digit) must be zero.
$\therefore$ $653xy = 653x0$, where $653x0$ must be divisible by 16 or $653x$ is divisible by 8.
Thus the last 3-digit number 53x will be divisible by 8.
Hence, at $x = 6$, we get the required result.
$\therefore$ $x + y = 6 + 0 = 6$

37. (c) In the given range, the last number which is divisible by both 5 and 7. i.e., 35 is 210 and the highest number is 770. So the total number of numbers between 200 and 800 which are divisible by both 5 and 7 is
$\left(\frac{770 - 210}{35}\right) + 1 = 17$
Hence option (c) is correct.

38. (a) Total numbers in the set = (800 – 200) + 1 = 601

Number of numbers which are divisible by 5

$$=\frac{(800-200)}{5}+1=121$$

Number of numbers which are divisible by 7

$$=\frac{(798-203)}{7}+1=86$$

Number of numbers which are divisible by both 5 & 7

$$=\frac{(770-210)}{35}+1=17$$

∴ Number of numbers which are either divisible by 5 or 7 or both

= (121 + 86) – 17 = 190

39. (d) Since Dividend = Divisor × Quotient + Remainder

∴ Dividend = 9235 × 888 + 222

Thus the number = 8200902

Hence (d) is the correct choice.

40. (c) Let this number be N then

$N = 32 \times Q_1 + 29$ ...(1)

Again $N = 8 \times Q_2 + R$ ...(2)

From (1) and (2)

$32Q_1 + 29 = 8Q_2 + R$ (where R is the remainder)

$8Q_2 - 32Q_1 = 29 - R$

$8(Q_2 - 4Q_1) = 29 - R$

or$(Q_2 - 4Q_1) = \frac{29-R}{8}$

Since Q_1, Q_2, R are integers also $Q_2 - 4Q_1$ is an integer.

Therefore 29 – R must be divisible by 8.

41. (d) $(0.\bar{1})^2\left[1-9(0.\overline{16})^2\right]$

$$=\left(\frac{1}{9}\right)^2\left[1-9\times\left(\frac{16}{99}\right)^2\right]$$

$$=\frac{1}{81}\left[1-9\times\frac{256}{9801}\right]$$

$$=\frac{1}{81}\left[1-\frac{256}{1089}\right]=\frac{1}{81}\times\frac{833}{1089}=\frac{833}{88209}$$

42. (d) Since the 7, 11 and 13 all are the factors of such a number so (d) is the correct answer.

43. (c) ∵ $7056 = 2^4 \times 3^2 \times 7^2$

∴ Number of factors/divisors of 7056

Product of factors = $(7056)^{45/2} = (84)^{45}$

Hence (c) is the correct option.

44. (d) The sum of digits of the number will be 114, which leaves a remainder of 6 when divided by 9. So when divided by 18 it would leave either 6 or 6 + 9 = 15 as the remainder.

Since the number is odd, it will leave an odd remainder only when divided by 18. So the remainder will be 15.

45. (a) $\frac{a+220}{a+4}=\frac{a+4+216}{a+4}=1+\frac{216}{a+4}$

Therefore, $(a + 4)$ must be a factor of 216.

The number of factors of 216 = 16

But $(a + 4)$ cannot be equal to 1, 2, 3 and 4 as 'a' has to be a positive integer.

Total possible values = 16 – 4 = 12

46. (a) Sum of all even factors:

$(2^1)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2) = 3534$

Number of even factors = 1 × 3 × 3 = 9

47. (d) Sum of divisors of 544 which are perfect square is:

$(2^0 + 2^2 + 2^4)(17^0) = 21$.

48. (c) Count the number of fives. This can get done by:

$100^1 \times 95^6 \times 90^{11} \times 85^{16} \times 80^{21} \times 75^{26} \times \ldots 5^{96}$

(1 + 6 + 11+ 16+ 21 +26 + 31 + 36 + 41 + 46 + ... + 96) + (1 + 26 + 51 + 76)

= 20 × 48.5 + 4 × 38.5 (Using sum of A.P. explained in the next chapter.)

= 970 + 154 = 1124.

49. (a) $(23)_5 = (2 \times 5^1 \times 3 \times 5^0)_{10} = (13)_{10} = (1 \times 8^1 + 5 \times 8^0)_8$

$= (15)_8$

also, $(47)_9 = (4 \times 9^1 + 7 \times 9^0)_{10} = (43)_{10}$

$= (5 \times 8^1 + 3 \times 8^0) = (53)_8$

sum $= (13)_{10} + (43)_{10} = (56)_{10} \rightarrow (70)_8$

50. (b) If we look at the numbers $100 < N \le 105$, we see only 101 and 103 do not have their factors in N (because these are primes). So, obviously the new LCM will be $101 \times 103 \times N$.

51. (c) The number needs to be less than 13 × 52 = 676. The highest power of 13 in 676! is 56.

The power of 13 in the smallest such number needs to be exactly 52. If we subtract 13 × 3 = 39 from 676, we get 637. The number 637! will be the smallest number of type N! that is completely divisible by 1352.

The sum of the digits of 637 is 16.

52. (d) $12^{55}/3^{11} = 3^{44}.4^{55} \rightarrow 4$ as units place.

Similarly, $8^{48}/16^{18} = 2^{72} \rightarrow 6$ as the units place.

Hence, 0 is the answer.

53. (c) It can be seen that the first expression is larger than the second one. Hence, the required answer would be given by the (units digit of the first expression – units digit of the second expression) = 6 – 0 = 6.

54. (c) The given numbers are two consecutive even numbers, so their HCF = 2

Now, using LCM × HCF = Product of two numbers

LCM × 2 = (…6) × (…8)

It can be seen now that the unit digit of LCM = 4

55. **(a)** Answer is LCM of 40, 42, 45 = $2^3 \times 3^2 \times 5^1 \times 7^1$ = 2520 cm = 25.2 m.

56. **(b)** Go through option

$S_n = 1 + 3 + 5 + 7 + \ldots + 22221$

$S_{11111} = (11111)^2$

Hence it is divisible by 11111. Thus option (b) is correct.

57. **(b)** $(50^2 - 1) = (50 + 1)(50 - 1) = (17 \times 3) \times (7 \times 7)$ hence divisible by 17.

and $(729)^5 - 729 = 729\,(729^4 - 1)$

$= 729\,(729^2 - 1)\,(729^2 + 1)$

$= (729)\,(729 - 1)\,(729 + 1)\,(729^2 + 1)$

$= 729 \times 728 \times 730 \times (729^2 + 1)$

Hence it is divisible by 5.

58. **(b)** Since $\frac{6}{10} \rightarrow$ Remainder is 6

$\frac{6^6}{10} \rightarrow$ Remainder is 6

$\frac{6^{6^6}}{10} \rightarrow$ Remainder is 6

59. **(b)** The answer will be 50 since, 125*122 will give 50 as the last two digits.

60. **(a)** $(2^{100} - 1)$ and $(2^{120} - 1)$ will yield the *GCD* as $2^{20} - 1$.

61. **(c)** Let us assume that the quotient is Q and divisor is D. Using the condition given in question, 1997 = QD + 41

$\Rightarrow$ QD = 1956. Now we will factorize 1956 in two parts such that D (divisor) is more than 41.

62. **(a)** 6^n (where n is a natural number) will always leaves the remainder 6 when divide by 10.

63. **(c)** For any *n*, 199^{2n} has last digit as 1,

But the last digit of 144^{3n} is 4 for odd values of *n* and 6 for even values of n.

Therefore, last digit of the given expression is either 5 or 7.

64. **(c)** Required number = H.C.F of (140 – 4), (176 – 6) and (264 – 9) = H.C.F. of 136, 170 and 255.

```
136)255(1
    136
    119)136(1
        119
        17)119(7
           119
            ×

17)170(10
   17
    0
```

$\therefore$ Required number = 17

65. **(b)** If the numbers be $3x$ and $4x$, then

HCF = x = 5

$\therefore$ Number = 15 and 20

$\therefore$ LCM = 12x = 12 × 5 = 60

66. **(a)** H.C.F. of 403, 434 and 465 is 31.

67. **(c)** LCM of $\frac{2}{3}, \frac{4}{9}, \frac{5}{6}$

$$\frac{\text{LCM of}(2,4,5)}{\text{HCF of }(3,9,6)} = \frac{20}{3}$$

68. **(d)** $(100x + 10y + z) - (x + y + z) = 99x + 9y$

$= 9\,(11x + y)$

69. **(a)** Let the number be x.

Then, $x - 31 = 75 - x$

$2x = 106$

$x = 53$

70. **(c)** First number = 2 × 44 = 88

Other number $= \frac{44 \times 264}{88} = 132$

71. **(c)** Third number $= \frac{265}{5} = 53$

$\therefore$ Smallest number = 49

Largest number = 57

$\therefore$ Required value

= 57 + 2 × 49

= 57 + 98 = 155

72. **(d)** Let A = x,

B = x + 2,

C = x + 4

$\therefore$ According to the question

$4x = 3(x + 4)$

$\Rightarrow$ $4x - 3x = 12 \Rightarrow x = 12$

$\therefore$ B = x + 2 = 12 + 2 = 14