1. (c) $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25.$
$= (\log 10/\log 5) \times (\log 15/\log 10) \times (\log 20/\log 15) \times (\log 25/\log 20)$
$= \log 25/\log 5 = 2 \log 5/\log 5 = 2.$

2. (d) $\because \log_3 a = 4 \therefore 3^4 = a \Rightarrow a = 81$

3. (a) Given $\log\frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4} = \log\left(\frac{9}{8} \div \frac{27}{32}\right) + \log\frac{3}{4}$
$= \log\left(\frac{9}{8} \times \frac{32}{27} \times \frac{3}{4}\right) = \log 1 = 0 \quad \because \log_a 1 = 0$

4. (a) Given : $3^{2-\log_3 5} = 3^2 . 3^{-\log_3 5}$ $(\because a^{m+n} = a^m . a^n)$
$= 9.3^{\log_3 5^{-1}} = 9 \times 5^{-1} = \frac{9}{5}$

5. (b) Given expression
$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$
$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$
$= 2 \log_{xyz}(xyz) = 2 \times 1 = 2$

6. (a) $\log_2 [\log_3 (\log_2 x)] = 1 = \log_2 2$
$\Rightarrow \log_3 (\log_2 x) = 2$
$\Rightarrow \log_2 x = 3^2 = 9$
$\Rightarrow x = 2^9 = 512$

7. (a) Let $\log_{27}\left(\frac{1}{81}\right) = x$
$\therefore (27)^x = \frac{1}{81}$
$\therefore 3^{3x} = 3^{-4} \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$

8. (b) $\frac{8\log_8 8}{2\log_{\sqrt{8}} 8} = \frac{8 \times 1}{2\log_{\sqrt{8}}\left(\sqrt{8}\right)^2} = \frac{8}{4\log_{\sqrt{8}}\sqrt{8}} = \frac{8}{4} = 2$

9. (a) $\log_3 (5 + x) + \log_8 8 = 2^2$
$\log_3 (5 + x) + 1 = 4$
$\log_3 (5 + x) = 3$
$3^3 = 5 + x$
$5 + x = 27$
$x = 27 - 5 = 22.$

10. (c) $\log_6 216\sqrt{6} = \log_6 (6)^3 (6)^{1/2} = \log_6 (6)^{7/2}$
$= \frac{7}{2}\log_6 6 = \frac{7}{2}$ $(\because \log_a a = 1)$

11. (d) Given, $\log_5 k \log_k x = 3$
$\frac{\log k}{\log 5} \cdot \frac{\log x}{\log k} = 3 \Rightarrow \frac{\log x}{\log 5} = 3$
$\Rightarrow \log x = 3 \log 5 \Rightarrow \log x = \log 5^3$
$\Rightarrow x = 5^3 \Rightarrow x = 125$

12. (b) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

13. (c) $\log_5\left[\log_3\left(\log_2 x\right)\right] = 1 = \log_5 5$
$\Rightarrow \log_3\left(\log_2 x\right) = 5 = \log_3 3^5$
$\Rightarrow \log_2 x = 3^5 = 243$
$\Rightarrow 2^{243} = x$

14. (d) $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$
$= \log\left[\left(\frac{81}{80}\right)^3 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{16}{15}\right)^7\right]$
$= \log\left(\frac{3^{12} \times 5^{10} \times 2^{28}}{2^{12} \times 5^3 \times 2^{15} \times 3^5 \times 3^7 \times 5^7}\right)$
$= \log 2$

15. (c) $\log_{10} a + \log_{10} b = c$
$\Rightarrow \log_{10}(ab) = c$
$\Rightarrow 10^c = ab$
$\Rightarrow a = \frac{(10)^c}{b}$

16. (d) $\log_y x = 8 \Rightarrow y^8 = x$ ...(1)
$\log_{10y} 16x = 4 \Rightarrow 10^4 y^4 = 16x$ ...(2)
Dividing (2) by (1) $10^4 y^{-4} = 16 \Rightarrow y = 5$

17. (b) $\log 0.0867 = \log (8.67/100) = \log 8.67 - \log 100$
$\log 8.67 - 2$

18. (d) $x = \log_{0.01} 2 = -\log 2/2.$

19. (b) $2^x . 3^{2x} = 100$
$\Rightarrow x \log 2 + 2x \log 3 = \log 100$
$\Rightarrow x(0.3010 + 2 \times 0.4771) = 2$
$\Rightarrow x = \frac{1}{1.2552} = 1.59$

20. (a) $\log_{10} a = b \Rightarrow 10^b = a \Rightarrow$ By definition of logs.
Thus $10^{3b} = (10^b)^3 = a^3.$

21. (b) $2 \log (81/100) \times 2/3 \log (27/10) \div \log 9$
$= 2 [\log 3^4 - \log 100] \times 2/3 [(\log 3^3 - \log 10)] \div 2 \log 3$
$= 2 [\log 3^4 - \log 100] \times 2/3 [(3 \log 3 - 1)] \div 2 \log 3$
Substitute $\log 3 = 0.4771 \Rightarrow -0.0552$

22. (d) $\log_{10} 10 + \log_{10} 10^2 + \ldots\ldots + \log_{10} 10^n$
$= 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

23. (a) $\log_b a \log_c a + \log_a b \log_a b + \log_a c \log_b c = 3$

$$\Rightarrow \frac{(\log a)^2}{\log b.\log c} + \frac{(\log b)^2}{\log a \log c} + \frac{(\log c)^2}{\log a \log b} = 3$$

$$\Rightarrow \frac{(\log a)^3 + (\log b)^3 + (\log c)^3}{\log a.\log b.\log c} = 3$$

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a.\log b.\log c$

We know, $x^3 + y^3 = z^3 = 3xyz$ when $x + y + z = 0$

$\log a + \log b + \log c = 0 \Rightarrow \log abc = 0 \Rightarrow abc = 1$.

PART - II

1. (a) $\log_{10} 80 = \log_{10}(8 \times 10) = \log_{10} 8 + \log_{10} 10$

$= \log_{10} 2^3 + 1$

$= (3 \log_{10} 2) + 1 = (3 \times 0.3010) + 1 = 1.9030$

2. (c) Let $\log_{2\sqrt{3}}(1728) = x$.

Then, $(2\sqrt{3})^x = 1728 = (12)^3$

$= \left[(2\sqrt{3})^2\right]^3 = (2\sqrt{3})^6$.

$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}}(1728) = 6$.

3. (b) $\log 4^{50} = 50 \log 4 = 50 \log 2^2$

$= (50 \times 2) \log 2 = 100 \times \log 2$

$= (100 \times 0.30103) = 30.103$

So the number of digits = 31.

4. (b) $\log_7 \log_5(\sqrt{x} + 5 + \sqrt{x}) = 0$

use $\log_a x = b$

$\Rightarrow a^b = x$

$\therefore \log_5(\sqrt{x} + 5 + \sqrt{x}) = 7^0 = 1$

$\sqrt{x} + 5 + \sqrt{x} = 5^1 = 5 \Rightarrow 2\sqrt{x} = 0 \therefore x = 0$

5. (d) Consider $\log_3[\log_3[\log_3 x]] = \log_3 3$

$\Rightarrow \log_3[\log_3 x] = 3$

$\Rightarrow \log_3 x = 3^3$

$\Rightarrow \log_3 x = 27 \Rightarrow x = 3^{27}$

6. (b) Let $\log(a + \sqrt{a^2+1}) + \log\left(\frac{1}{a + \sqrt{a^2+1}}\right)$

$= \log(a + \sqrt{a^2+1}) + \log 1 - \log(a + \sqrt{a^2+1})$

$= \log(a + \sqrt{a^2+1}) - \log(a + \sqrt{a^2+1})$

$= 0$

7. (a) $\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ac)+1} + \frac{1}{(\log_c ab)+1}$

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ac + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$\log_{abc} abc = 1$

8. (c) $q = \log_{17}(5)^2 = 2\log_{17} 5$

$\Rightarrow \frac{1}{q} = \frac{1}{2}\log_5 17$

And $\frac{1}{p} = \log_5 3 = \frac{1}{2} \times (2\log_5 3)$

$= \frac{1}{2}(\log_5 9)$

$\therefore \frac{1}{p} < \frac{1}{q} \Rightarrow p > q$

9. (c) $(pa + qb - rc) = p\log_{10} x + q\log_{10} y - r\log_{10} z$

$= \log_{10}(x^p) + \log_{10}(y^q) - \log_{10}(z^r)$

$= \log_{10}\left(\frac{x^p y^q}{z^r}\right)$

$\Rightarrow$ antilog $(pa + qb - rc) = \frac{x^p y^q}{z^r}$

10. (c) a, b, c are consecutive integers

$\therefore b = a + 1$ and $c = a + 2$

$\therefore \log(ac + 1) = \log[a(a + 2) + 1]$

$= \log[(b - 1)(b - 1 + 2) + 1]$ $[\because a = b - 1]$

$= \log b^2 = 2\log b$

11. (c) $(7^3)^{-2\log_7 8} = 7^{-6\log_7 8} = 7^{(\log_7 8^{-6})}$

$= 8^{-6} = \frac{1}{8^6}$

12. (b) Given equation is $(\log_3 x)^2 + \log_3 x < 2$

$\Rightarrow (\log_3 x)^2 + (\log_3 x) - 2 < 0$

$\Rightarrow (\log_3 x + 2)(\log_3 x - 1) < 0$

$\Rightarrow -2 < \log_3 x < 1$

$\Rightarrow \log_3 3^{-2} < \log_3 x < \log_3 3$

$\Rightarrow \frac{1}{9} < x < 3$

13. (b) Let $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$

$\Rightarrow \frac{1}{2}\log_{10} x = 2\log_x 10 \Rightarrow \log_{10} x = \log_x 10^4$

$\Rightarrow \frac{\log_{10} x}{\log_x 10} = 4 \Rightarrow (\log_{10} x)^2 = 4$

$\Rightarrow \log_{10} x = \pm 2$

$\Rightarrow x = 10^2$ or 10^{-2}

14. (d) The given logarithm expression

$$\frac{\log_{27} 9 \log_{16} 64}{\log_4 \sqrt{2}}$$

is simplified as :

$$\frac{\log 9}{\log 27}\times\frac{\log 64}{\log 16}\times\frac{\log 4}{\log\sqrt{2}}$$

$$=\frac{2\log 3}{3\log 3}\times\frac{6\log 2}{4\log 2}\times\frac{2\log 2}{\frac{1}{2}\log 2}$$

$$=\frac{2}{3}\times\frac{6}{4}\times 4=4$$

15. (b) $(\log_x x)(\log_3 2x)(\log_{2x} y)=\log_x x^2$

$\Rightarrow 1(\log_3 2x)(\log_{2x} y)=2 \quad (\because \log_x x^2)=2\log_x x)$

$$\Rightarrow\left(\frac{\log 2x}{\log 3}\right)\left(\frac{\log y}{\log 2x}\right)=2$$

$$\Rightarrow \frac{\log y}{\log 3}=2 \Rightarrow \log y = 2\log 3$$

$\Rightarrow \log y=\log 3^2 \Rightarrow y=3^2 \Rightarrow y=9$

16. (d) Consider, $\log\frac{9}{8}-\log\frac{27}{32}+\log\frac{3}{4}$

$$=\log\left(\frac{9}{8}\times\frac{32}{27}\right)+\log\frac{3}{4}$$

$$=\log\left(\frac{4}{3}\right)+\log\frac{3}{4}=\log\left(\frac{4}{3}\times\frac{3}{4}\right)=\log 1=0$$

17. (b) $\log_{10}x$, $\log_{10}y$, $\log_{10}z$ are in AP

$\therefore \quad 2\log_{10} y=\log_{10} x+\log_{10} z$

$\Rightarrow \quad \log_{10} y^2=\log_{10}(xz)$

$\Rightarrow \quad y^2=xz$

$\Rightarrow \quad x, y, z$ are in GP

18. (c) $$\frac{\log\sqrt{27}+\log\sqrt{8}-\log\sqrt{125}}{\log 6-\log 5}$$

$$=\frac{\log\left(3^{3/2}\times 2^{3/2}\right)-\log(5)^{3/2}}{\log 6-\log 5}$$

$$=\frac{\log 6^{3/2}-\log 5^{3/2}}{\log 6-\log 5}$$

$$=\frac{\frac{3}{2}\log\left(\frac{6}{5}\right)}{\log\left(\frac{6}{5}\right)}=\frac{3}{2}$$

19. (b) Best way is to go through options

Alternatively: $\log_{10} x^2y^3=7$

$\Rightarrow \quad x^2y^3=10^7 \qquad$ ...(1)

and $\log_{10}\left(\frac{x}{y}\right)=1$

$\Rightarrow \quad \frac{x}{y}=10 \qquad$ ...(2)

$$\therefore \quad \frac{x^2y^3}{(x/y)^2}=\frac{10^7}{(10)^2}\Rightarrow y^5=10^5$$

$\Rightarrow \quad y=10 \therefore x=100$

20. (d) $\log_3 2, \log_4 3. \log_5 4....\log_{16} 15$

$$=\frac{\log 2}{\log 3}\cdot\frac{\log 3}{\log 4}\cdot\frac{\log 4}{\log 5}\cdots\cdots\frac{\log 15}{\log 16}=\frac{\log 2}{\log 16}=\frac{\log 2}{\log 2^4}$$

$$=\frac{\log 2}{4\log 2}=\frac{1}{4}$$

21. (d) $\log_4 5=a$ and $\log_5 6=b$

$\Rightarrow \log_4 5\times\log_5 6=ab$

$\Rightarrow \log_4 6=ab \Rightarrow \frac{1}{2}\log_2 6=ab$

$\Rightarrow (1+\log_2 3)=2ab$

22. (b) $\log_3 x+\log_9 x+\log_{27} x+\log_{81} x=\frac{25}{4}$

$$\Rightarrow \log_3 x+\frac{1}{2}\log_3 x+\frac{1}{3}\log_3 x+\frac{1}{4}\log_3 x=\frac{25}{4}$$

$$\Rightarrow \log_3 x\,[1+1/2+1/3+1/4]=\frac{25}{4}$$

$$\Rightarrow \frac{25}{2}\times\log_3 x=\frac{25}{4}$$

$\Rightarrow \log_3 x=3 \Rightarrow x=27$

23. (d) $\log_{32} 27\times\log_{243} 8=\log_{2^5} 3^3\times\log_{3^5} 2^3$

$$=\frac{3}{5}\log_2 3\times\frac{3}{5}\log_3 2$$

$$=\left(\frac{3}{5}\right)^2\log_2 3\times\log_3 2=\frac{9}{25}$$

24. (c) $\log\left(a^nb^nc^n/a^nb^nc^n\right)=\log 1=0$